Correspondence

Some Effects of Dielectric Loading on Ferrite Phase Shifters in Rectangular Waveguide*

Introduction

The use of dielectric loading with the ferrites in nonreciprocal transmission lines has become quite prevalent. The theoretical analysis of unloaded ferrite phase shifters in rectangular waveguide[1] has been extended to the dielectrically loaded case by Soohoo.[2] The boundary-value problem of ferrite phase shifters in dielectrically loaded coaxial lines, which possesses a transcendental equation similar to the waveguide case, has been throughly discussed by several authors.[3–5] The present communication reports the solution of the boundary-value problem of a dielectrically loaded ferrite phase shifter in rectangular waveguide to yield values of the lossless propagation constant for various slab positions for both directions of propagation. The theoretical values of the differential phase shift for slab positions about the center of the waveguide were compared with the measured values for three different applied magnetic fields.

The purpose of the dielectric loading is to concentrate the RF fields in the region of the ferrite slab. The amount of coupling of the RF h-field into the ferrite, and, thereby, the amount of phase shift, will be increased for one direction of propagation. If the dielectric loading is not excessive, increased differential phase shifts without anomalous propagation effects should be obtained for slab positions near the center of the waveguide.[6] Values of the differential phase shift, calculated by a method similar to that of Lax, Button, and Roth,[1] might then be compared with the measured values.

The presence of a ferrite slab of finite thickness will grossly perturb the position of the plane of circularly polarized h-field in rectangular waveguide.[7] The amount of phase shift will thus be quite sensitive to the position of a ferrite slab in the waveguide. The dielectric loading, however, will cause a constant displacement of the RF fields into the ferrite for all slab positions in the center of the waveguide. The sensitivity of the differential phase shift to slab position will thus be reduced.

Theoretical Analysis

A dielectric slab was placed parallel and adjacent to a ferrite slab extending the full height of a rectangular waveguide, as shown in Fig. 1. For the purposes of this analysis, it was assumed that the dominant rectangular waveguide mode was propagating. Application of the boundary conditions at each of the three interfaces led to a transcendental equation in d, the position of the slab configuration:

$$-p \sin [k_a(L - t_1 - t_2 - 2d)] + q \cos [k_a(L - t_1 - t_2 - 2d)] + r = 0, \quad (1)$$

where

$$p = k_a\left[\left(\frac{\beta}{\theta}\right)^2 - k_m^2 + \left(\frac{k_d}{\rho}\right)^2\right] - 2j\frac{\beta}{\rho\theta} k_a k_d \cot (k_d t_2), \quad (2)$$

$$q = k_m \cot (k_m t_1)(k_a^2 - k_d^2) + j\frac{\beta\rho}{\theta}(k_a^2 + k_d^2) + k_d\left[\left(\frac{\beta}{\theta}\right)^2 - k_m^2 + \left(\frac{k_a}{\rho}\right)^2\right] \cdot \cot (k_d t_2), \quad (3)$$

$$r = \sin [k_a(L - t_1 - t_2)] \cdot \left\{k_a\left[\left(\frac{\beta}{\theta}\right)^2 - k_m^2 - \left(\frac{k_d}{\rho}\right)^2\right] + 2\frac{k_m}{\rho} k_a k_d \cot (k_m t_1) \cot (k_d t_2)\right\} + \cos [k_a(L - t_1 - t_2)] \cdot \left\{(k_a^2 + k_d^2)\frac{k_m}{\rho} \cot (k_m t_1) + j\frac{\beta}{\rho\theta}(k_a^2 - k_d^2) - k_d\left[\left(\frac{\beta}{\theta}\right)^2 - k_m^2 - \left(\frac{k_a}{\rho}\right)^2\right] \cdot \cos (k_d t_2)\right\}, \quad (4)$$

and where

$$k_a = \frac{\omega^2}{c^2} - \beta^2$$

$$k_d = \frac{\omega^2}{c^2} K_d - \beta^2,$$

$$k_m = \frac{\omega^2}{c^2}\frac{K_m}{\rho} - \beta^2,$$

$$\rho = \frac{1 + X_{xx}}{(1 + X_{xx})^2 + X_{xy}^2},$$

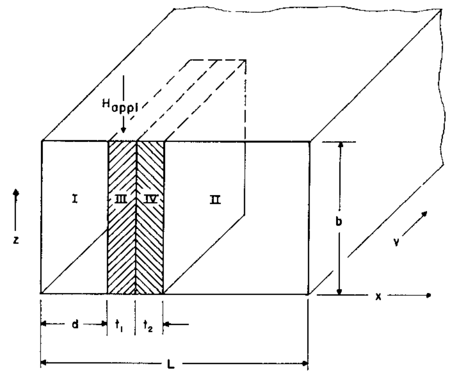

Fig. 1—Geometry of ferrite-dielectric slab configuration in rectangular waveguide. Regions I and II are air, Region III is the ferrite, and Region IV is the dielectric.

$$\theta = \frac{1 + X_{xx}}{X_{xy}},$$

$$X_{xx} = \frac{4\pi M_s \gamma \omega_0}{\omega_0^2 - \omega^2},$$

$$X_{xy} = -j\frac{4\pi M_s \gamma \omega}{\omega_0^2 - \omega^2},$$

$4\pi M_s$ = saturation magnetization of ferrite,

$\gamma = g\frac{e}{2mc}$ = gyromagnetic ratio of ferrite,

$\omega = 2\pi f$ = operating frequency,

$\omega_0 = \gamma(B_z H_z)^{1/2}$ = resonant frequency of ferrite,

$B_z = H_z + 4\pi M_s$ = magnetic induction,

H_z = applied static magnetic field,

$K_d = \frac{\epsilon_2}{\epsilon_0}$ = relative dielectric constant of dielectric slab,

$K_m = \frac{\epsilon_1}{\epsilon_2}$ = relative dielectric constant of ferrite slab, and

c = velocity of light in vacuo.

The transcendental equation (1) may be solved numerically to yield values of the slab position d for given values of the phase constant, β. Since (1) is actually quadratic in cos $[k_a (L - t_1 - t_2 - 2d)]$, there will be, in general, two valid values of the slab position for each value of the phase constant, one for each direction of propagation. The theoretical value of the differential phase shift is defined as

$$\Delta\beta \equiv \beta_+ - \beta_-, \quad (5)$$

where

β_+ = the computed phase constant of the waveguide partially filled with the ferrite-dielectric slab configuration

* Received by the PGMTT, August 24, 1961; revised manuscript received, September 8, 1961. The work reported in this communication formed the basis for a thesis presented to the Physics Department of the University of Pennsylvania in partial fulfillment of the requirements for the M.S. degree. This work was supported by the Philco Corporation Research Division.

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and propagating TE-mode waves in the positive y direction, and

β_- = the computed constant for TE-mode waves propagating in the negative y direction.

A typical plot of the computed phase constants vs slab position is shown in Fig. 2; the corresponding plot of the differential phase shift is shown in Fig. 3.

The values of the pertinent parameters were

microwave frequency,	$f = 16.0$ Gc,
inner width of K_u band waveguide,	$L = 0.622$ in,
g factor of ferrite,	$g = 2.2$,
relative dielectric constant of dielectric,	$K_d = 12.0$,
relative dielectric constant of ferrite,	$K_m = 10.0$,
saturation magnetization of ferrite,	$4\pi M_s = 1750$ Gauss.

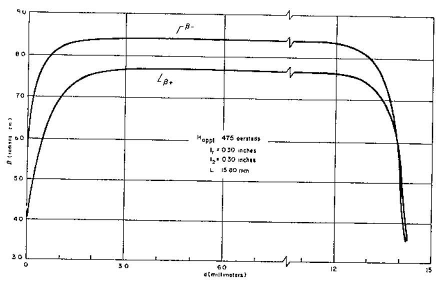

Fig. 2—Computed phase constants, β_+, vs slab position, d.

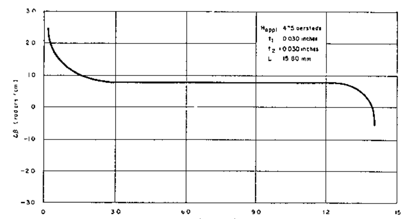

Fig. 3—Computed differential phase shift, $\Delta\beta$, vs slab position, d.

Experimental Data

Direct comparison of the experimental with the theoretical values of the differential phase shifts was obtained for a series of slab positions in the waveguide. The top and bottom broad walls of a section of waveguide were cut away and replaced by sliding metal plates. The narrow walls remained stationary. The ferrite-dielectric slab configuration was affixed to the plates and was moved so that the slabs were always parallel to the narrow walls. The phase shifts and insertion losses were measured by balancing a known electrical length of transmission line against the unknown length containing the ferrite-dielectric configuration.

The values of the differential phase shifts were measured for a series of slab positions for static fields of 475, 600, and 1000 oe. A typical graph of the measured differential phase shift vs slab position is shown in Fig. 4. As was to be expected, propagation was quite lossy for slab positions near the narrow walls of the waveguide,[6] and measurements of the phase shifts proved erratic.

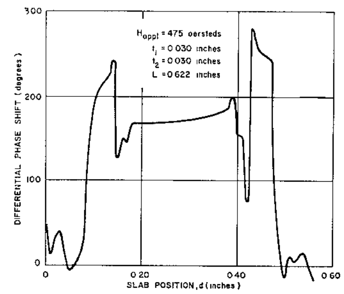

Fig. 4—Measured differential phase shift vs slab position.

The computed and measured values of the differential phase shifts are compared in Table I for the three values of the applied magnetic field. A constant, appreciable value of the differential phase shift is obtained for slab positions about the center of the waveguide. In contrast, the value of the differential phase shift for unloaded ferrite slabs tends toward zero[1] for central slab positions.

TABLE I
Comparison of Experimental and Theoretical Values of the Differential Phase Shift*

Applied Magnetic Field, H_z (oersteds)	Total Differential Phase Shift (experimental) (degrees)	(theoretical) (degrees)
475	170	169
600	176	180
1000	184	194

* d = 0.300 in = distance from waveguide wall.
l = 1.550 in = effective length of slab.

Acknowledgment

The author wishes to express his appreciation to Dr. M. E. Caspari for his advice and counsel in this work, to W. S. Koop for his valuable guidance, and to M. N. Schleifer for his capable programming of the numerical calculations.

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The Gyromagnetic Coupling Limiter at C-Band*

The original paper of DeGrasse[1] and subsequent publications dealing with the use of the crossed-strip gyromagnetic coupler as a limiter[2] have been concerned with the operation of this device below 3300 Mc. At these frequencies the limiter exhibits a sharp threshold at a very low power level, in the neighborhood of −20 dbm. Fig. 1 illustrates typical flat leakage characteristics at a frequency of 2600 Mc. A single crystal YIG sphere of 26 mils was used in this limiter.

When spherical single crystal YIG ($4\pi M_s = 1750$) resonators are used, the theory developed principally by Suhl[3] indicates that a pronounced change in limiting characteristics should occur at about 3300 Mc. Above this frequency the first-order nonlinear process[4] is forbidden at resonance and the limiting characteristics observed must be attributed to the second-order process.

A crossed-strip limiter constructed for operation in C-band employing a single crystal YIG sphere 23 mils in diameter with $\Delta H \approx 0.43$ oersted exhibits the flat leakage characteristics shown in Fig. 2. In contrast with the S-band limiter a relatively high threshold is apparent, and the shape of the limiting curve indicates the smooth spin wave excitation to be expected as a result of inhomogeneity broading.[4–6]

With no field applied to the YIG sphere a strip-to-strip isolation slightly in excess of

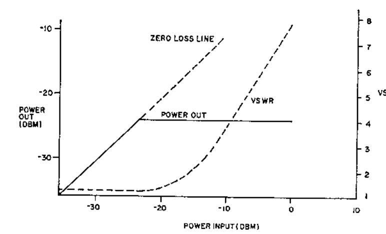

Fig. 1—Typical limiting characteristics of a gyromagnetic coupler when operating with a YIG sphere at frequencies between 1600 and 3300.

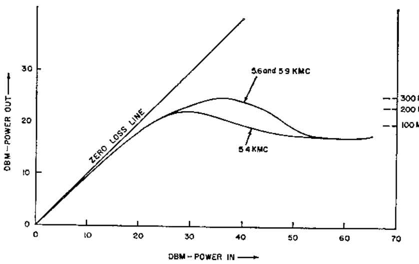

Fig. 2—Flat leakage characteristics of the C-band gyromagnetic coupling limiter.

* Received by the PGMTT, September 8, 1961. This work was supported in part by the U. S. Army Signal Res. and Dev. Lab. under Contract No. DA-36-039-SC-85330.

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